Worksheet 10 Two-Way ANOVA

Let’s perform a two-way ANOVA on the viscosity data we looked at in class, first in Excel and then in R.

Recall, an experiment was carried out to investigate the effect of pH and Catalyst Concentration on the Viscosity of a product. Three replicates of a design were performed, and the results are available on Blackboard in the viscosity.txt file.

To determine whether pH and Catalyst Concentration affect the Viscosity of the product and whether the two factors interact, we will carry out a two-way ANOVA. Let Factor A represent pH and let Factor B represent Catalyst Concentration.

The model can be written as:

represents the viscosity of replicate *k* at level *i* of pH and at level *j* of Catalyst Concentration *i* = 1, 2, *j* = 1,2, *k* = 1,2,3

represents the treatment effect of pH at level *i*

represents the treatment effect of Catalyst Concentration at level *j*

represents the interaction effect and ∼ NID(0,1).

The hypotheses associated with this two-way ANOVA are:

H₀ : α₁ = α₂ = ... = αa = 0

HA : at least one αi ≠ 0

H₀ : β₁ = β₂ = .. .= βb=0

HA : at least one βj ≠ 0

H₀ : (αβ)ij = 0 for all *i,j*

HA : at least one (αβ)ij ≠ 0

We will test these hypothesis at the 5% significance level so α = 0.05

Import the data set into R and convert the variables pH and Catalyst to factors.

The table function will display how many observations there are in each treatment.

table(viscosity$Catalyst, viscosity$pH)

5.6 5.9

2.5 3 3

2.7 3 3

Visualise the effect of Catalyst Concentration on Viscocity:

boxplot(viscosity$Viscosity~viscosity$Catalyst, ylab = "Viscosity")

Visualise the effect of pH on Viscocity:

boxplot(viscosity$Viscosity~viscosity$pH, ylab = "Viscosity")

Create a boxplot showing the distribution of Viscosity for each treatment:

boxplot(Viscosity~Catalyst\*pH, data=viscosity, col=(c("gold","darkgreen")), main="Viscosity", xlab="Catalyst and pH")

Calculate the mean and variance of the Viscosity for each Catalyst level and each pH level.

viscosity %>% group\_by(Catalyst) %>%

summarise( mean\_viscosity = mean(Viscosity), var\_viscosity = var(Viscosity))

viscosity %>% group\_by(pH) %>%

summarise( mean\_viscosity = mean(Viscosity), var\_viscosity = var(Viscosity))

Check for an interaction between pH and Catalyst Concentration

interaction.plot(viscosity$pH, viscosity$Catalyst, viscosity$Viscosity)

Check homogeneity of variance

fligner.test(Viscosity~interaction(pH,Catalyst), data = viscosity)

What does the result of the test tell you?

Fit two way ANOVA with interaction effect, both calls are equivalent

model1<-aov(Viscosity~Catalyst\*pH, data = viscosity)

model2<-aov(Viscosity~Catalyst + pH +Catalyst:pH, data = viscosity)

summary(model1)

summary(model2)

Df Sum Sq Mean Sq F value (>F)

Catalyst 1 1.3 1.3 0.070 0.79778

pH 1 75.0 75.0 3.947 0.08218 .

Catalyst:pH 1 320.3 320.3 16.860 0.00341 \*\*

Residuals 8 152.0 19.0

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

It can be seen that the interaction between Catalyst and pH is statistically significant which indicates that the relationship between pH and Viscosity depends on the level of Catalyst Concentration. You could also say that the relationship between Catalyst Concentration and Viscosity depends on the level of pH. In this case, where there is such a strong interaction between the two factors, interpretation of the main effects is not meaningful. Instead we can examine the simple effects using TukeyHSD().

Aside: Once the ANOVA model has been fitted it is possible to calculate summary statistics automatically using the model.tables() function.

model.tables(model1, type="means", se=F)

Tables of means

Grand mean

191.6667

Catalyst

2.5 2.7

192.00 191.33

pH

5.6 5.9

189.17 194.17

Catalyst:pH

pH

Catalyst 5.6 5.9

2.5 194.67 189.33

2.7 183.67 199.00

Means across all levels of the factors and treatments are calculated.

Effect Size

To measure the effect size associated with a one way ANOVA, we use (Partial Eta squared).

For pH

or 33%. We can say that 33% of the variation in Viscosity was caused by pH.

For small sample sizes, can be biased and overestimate the proportion of variance for small sample sizes. An alternative measure is

For pH:

The sjstats package can calculate these measures for us:

install.packages("sjstats")

eta\_sq(model1, partial=T)

omega\_sq(model1)

Post –Hoc Test

The results of the ANOVA test, indicated that at least one pair of group means is different, but we don’t know which. To find out we can use Tukey’s HSD test.

TukeyHSD(model1)

Tukey multiple comparisons of means

95% family-wise confidence level

Fit: aov(formula = Viscosity ~ Catalyst \* pH, data = viscosity)

$Catalyst

diff lwr upr p adj

5.9-5.6 5 -0.8033165 10.80332 0.0821793

$pH

diff lwr upr p adj

2.7-2.5 -0.6666667 -6.469983 5.13665 0.7977791

$`Catalyst:pH`

diff lwr upr p adj

5.9:2.5-5.6:2.5 -5.333333 -16.730588 6.0639210 0.4808069

5.6:2.7-5.6:2.5 -11.000000 -22.397254 0.3972543 0.0585163

5.9:2.7-5.6:2.5 4.333333 -7.063921 15.7305877 0.6337900

5.6:2.7-5.9:2.5 -5.666667 -17.063921 5.7305877 0.4337337

5.9:2.7-5.9:2.5 9.666667 -1.730588 21.0639210 0.0993067

5.9:2.7-5.6:2.7 15.333333 3.936079 26.7305877 0.0110745

The results for the main effects of pH and Catalyst are not significant.

Examining the simple effects shows if catalyst concentration is held at 2.7, then the mean viscosity of the material produced when the pH is 5.9 is significantly greater than the mean viscosity of the material produced when the pH is 2.7.

No other effects are significant.

Model Validation

Next we check the assumptions of the aov model (this is referred to as model validation).

windows(10,10)

par(mfrow = c(2,2))

plot(model1)

There are no obvious outliers and the variance looks similar across treatments. The very small sample size means that it is not possible to assess normality.

Report results

The first section of the report should include:

* a summary of the hypotheses and the data (number of levels of each factor, sample size etc.)
* summary statistics and graphs (including group means across each level)
* interaction plot and interpretation
* test of homogeneity of variance

Results of the two way ANOVA can be reported as follows:

A two-way ANOVA was run on a sample of 12 observations to examine the effect of pH and catalyst concentration on the viscosity of a material. There was a significant interaction between the effects of pH and catalyst concentration on the level of viscosity F(1, 8) = 16.86, MSE = 152, p = .003, . Simple effects analysis showed that the viscosity of the material was significantly greater when a pH of 5.9 was used in comparison to a pH of 5.6 (p = 0.01) for a catalyst concentration of 2.7 but there were no differences between viscosity for different levels of pH when catalyst concentration was held at 2.5 (p = .48).

Include a table or plot showing the results of Tukey’s HSD test and a brief summary of the diagnostics. For example:

Analysis of the standardized residuals indicated that there were no outliers and that homogeneity of variance was satisfied. Assessment of normality was not possible due to the small sample size.

Example No Interaction

Recall the example from class where an experiment was run to investigate how the type of glass and the type of phosphorescent coating affects the brightness of a light bulb. The response variable is the current (in microamps) to obtain a specified brightness. The data is available on Blackboard in the light\_bulb.txt file

Import the data and run a two-way ANOVA on the data, don’t forget to:

* state the hypotheses
* summarise and visualise the data
* check the homogeneity of variance assumption
* create an interaction plot

The results from the ANOVA model indicate that the interaction term is not significant (we expect this result based on the interaction plot).

Df Sum Sq Mean Sq F value Pr(>F)

light$Glass 1 7351 7351 164.569 1.29e-06 \*\*

light$Phosphor 1 631 631 14.121 0.00556 \*\*

light$Glass:light$Phosphor 1 0 0 0.002 0.96661

Residuals 8 357 45

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

We can drop the interaction term from the model and interpret the main effects directly:

Df Sum Sq Mean Sq F value Pr(>F)

light\_bulb$Glass 1 7351 7351 185.10 2.63e-07 \*\*\*

light\_bulb$Phosphor 1 631 631 15.88 0.00318 \*\*

Residuals 9 357 40

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Both Glass Type and Phosphor explained a significant amount of the variation in Current (p < 0.001 and p = 0.003 respectively). Since both Glass and Phosphor both have just two levels the ANOVA table tells us that there is a significant difference between the mean Current for bulbs produced using Glass type 1 and 2 and there is a significant difference between the mean Current for bulbs produced using Phosphor type 1 and 2. The differences between treatment means can be examined using Tukey’s HSD test.:

Tukey multiple comparisons of means

95% family-wise confidence level

Fit: aov(formula = light\_bulb$Current ~ light\_bulb$Glass + light\_bulb$Phosphor)

$`light\_bulb$Glass`

diff lwr upr p adj

2-1 -49.5 -57.73054 -41.26946 3e-07

$`light\_bulb$Phosphor`

diff lwr upr p adj

2-1 14.5 6.269461 22.73054 0.0031802

Next we check the assumptions of the aov model

windows(10,10)

par(mfrow = c(2,2))

plot(model\_treat)

There are no obvious outliers and the variance looks similar across treatments. Again, the very small sample size means that it is not possible to assess normality.

Write up

A two-way ANOVA was run on a sample of 12 observations to examine the effect of glass type and phosphor type on the brightness of light bulbs where brightness was measured in microamps. There was no significant interaction between the effects of glass type and phosphor type on the current. Both main effects were significant. The current was significantly greater when glass type 1 was used F(1,9) = 185.1, MSE = 40 , p < 0.001, = 0.873 and when phosphor type 1 was used F(1,9) = 15.88, MSE = 40 , p = 0.003, = 0.071.

Exercise

An engineer is designing a battery for use in a device that will be subject to extreme variations in temperature. Three material types can be used to make the battery plates. The objective is to design a battery that is relatively unaffected by the ambient temperature. The output response from the battery is effective life in hours. Three temperature levels are selected, and a factorial experiment with four replicates is run. The data is available in the battery.txt file on Blackboard. Analyse the data using a two-way ANOVA and write a brief report stating the hypotheses associated with the test and any conclusions that you draw.